*J. Fluid Mech.* (1974), *vol.* 66, *part* 2, *pp.* 267–272
*Printed in Great Britain*


# Shear-flow stability within the atmosphere of Venus

By R. D. CESS AND HARSHVARDHAN
Department of Mechanics, State University of New York, Stony Brook

(Received 20 June 1973 and in revised form 12 February 1974)

Employing a linear stability analysis, Dudis (1973) has recently suggested that shear-flow instability might exist within the upper stratosphere of Venus owing to destabilization by radiative transfer. We have incorporated a more realistic formulation for radiative transfer into his stability analysis and conclude that such an instability is unlikely.

## 1. Introduction

An extremely interesting stability analysis has recently been presented by Dudis (1973) for a stratified, thermally radiating, unbounded shear layer. He considers the destabilizing influence of radiative transfer, whereas the corresponding non-radiating stability criterion is given by

$$Ri = (g/T)\,(dT/dz+\Gamma)/S^2 > \tfrac{1}{4}, \tag{1}$$

with $Ri$ denoting the Richardson number, $g$ the acceleration due to gravity, $dT/dz$ the vertical temperature gradient, $\Gamma$ the adiabatic lapse rate and $S = du/dz$ the vertical velocity gradient.

When radiative transfer is included, Dudis finds that the neutral-stability boundary can exhibit two maxima as illustrated in figure 1. One does not know *a priori* which of the two maxima dominates the stability problem; i.e. which of the two yields the larger Richardson number. The neutral-stability boundary and thus the values for $Ri_1$ and $Ri_2$ are shown by Dudis to depend upon the Reynolds number of the shear layer together with the dimensionless radiation parameter

$$G = \nabla . \mathbf{q}/\rho C_p S T', \tag{2}$$

where $\mathbf{q}$ and $T'$ represent perturbations to the radiative heat flux and temperature, respectively, while $\rho$ denotes density and $C_p$ the specific heat.

Dudis considers several atmospheric examples, and by employing the optically thin limit for radiative transfer, he estimates that the critical Richardson number at the 100 km level within the atmosphere of Venus exceeds the actual Richardson number, inferring that shear-flow instability exists within the atmosphere of Venus. Such a conclusion has strong implications concerning our understanding of mixing processes within the atmosphere of Venus, and we feel that it is useful to examine this conclusion employing a more realistic model for radiative transfer.

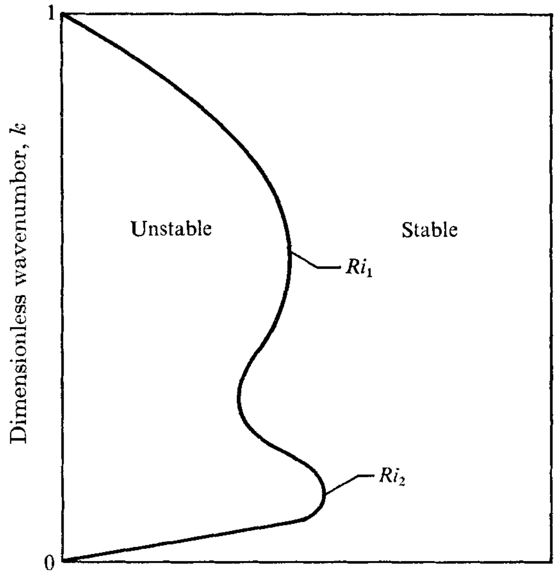


FIGURE 1. Illustrative neutral-stability boundary following Dudis (1973).

## 2. Length-scale considerations

As discussed by Dudis, his stability analysis in the non-radiating limit is equivalent to that of Maslowe & Thompson (1971), for which the static stability of the density stratification is expressed as

$$\lambda(z) \equiv \ln [\rho_0/\rho(z)] = (L/H) \tanh (z/L), \tag{3}$$

where, as will be discussed shortly, $L$ is the length scale of the shear layer, while $\rho_0$ denotes a reference density. For a hydrostatic atmosphere, on the other hand, $\lambda(z) = gz/RT$, with $R$ denoting the gas constant, and at least for $z/L \ll 1$, this is equivalent to (3) provided that $H$ is interpreted as the scale height of the atmosphere; i.e. $H = RT/g$.

The length scale $L$ which appears in Dudis' hyperbolic-tangent mean velocity profile is a length scale characteristic of the shear layer. It is difficult to choose a value for $L$. Dudis considers rather small values (200 m), but the interpretation of the Soviet Venera 7 data by Ainsworth & Herman (1972), admittedly referring to levels within the atmosphere of Venus lower than those considered here, implies a shear-layer thickness at least as great as the scale height, which is consistent with the arguments put forth by Gierasch, Goody & Stone (1970) for the vertical scale of fluid motions, as well as the analysis of Ramanathan (1973) concerning zonal flow within the stratosphere of Venus. We shall thus choose $L = H$, and further discussion on this point is given later.

We shall furthermore make direct use of Dudis' stability analysis, which employs the Boussinesq approximation. This is strictly valid only for motions whose vertical length scale is less than a scale height. But application of the Boussinesq approximation will tend to underestimate the static stability of the atmosphere, producing an upper bound on the critical Richardson number, which is sufficient for present considerations.

The final length scale pertains to radiation, radiative transport within the stratosphere being due to the 15 $\mu$m fundamental band of $CO_2$. Dudis, employing the previously mentioned small length scale $L = 200$ m for the shear layer, assumes that the shear layer is optically thin. He bases this assumption upon considerations of the Planck mean absorption coefficient, but this coefficient is extremely misleading with regard to the determination of optical thinness, since it constitutes a mean over the entire black-body spectrum.

A realistic criterion for optically thin radiation requires that the individual rotational lines of the 15 $\mu$m $CO_2$ band correspond to the weak-line limit, and from Cess & Tiwari (1972) this requires that $\delta \ll 1$ for Lorentz lines, where†

$$\delta = S_1 Ld/4\gamma_0 A_{01}, \tag{4}$$

with $\gamma_0$, $S_1$, $A_{01}$ and $d$ denoting, respectively, the mean line width per unit pressure, band intensity, bandwidth parameter and mean line spacing.

From (3), and for conditions appropriate to the stratosphere of Venus, we find that the weak-line (optically thin) requirement $\delta \ll 1$ corresponds to $L \ll 0{\cdot}01$ cm, which is a most unrealistic restriction.‡ Concerning a stability analysis, the length scale $L$ must be associated with the vertical length scale of a temperature perturbation, and Dudis has shown that this corresponds roughly to the shear-layer thickness, or by previous arguments, to the scale height of the atmosphere. Clearly $\delta \gg 1$ for realistic shear layers, such that the strong-line limit applies (Cess & Ramanathan 1972) rather than the weak-line limit. In the following section we present a radiation model which is appropriate for present purposes.

Before proceeding, however, it should be mentioned that quite often attempts are made to formulate analytically the radiative flux for a real gas by employing a modified grey-gas differential approximation as a device for interpolating between the optically thin and optically thick limits. But this is not valid for the problem at hand, since the optically thick limit does not exist for a vibration–rotation band (Cess, Mighdoll & Tiwari 1967), while the optically thin limit is not a relevant limit, and such a procedure in addition ignores the discrete line structure of the band.

## 3. Radiative-transfer formulation

As discussed by Cess & Ramanathan (1972), there exist, in addition to the strong-line limit, further asymptotic radiation limits, which are characterized by the strong-line parameter

$$\xi = (3S_1 \gamma_0 H/A_{01} d)P^2, \tag{5}$$

† This definition for $\delta$ corresponds to a homogeneous layer of thickness $L$, and in an atmospheric context this implies that $L \ll H$. For $L = H$, use of the Curtis–Godson approximation replaces the 4 by 2 in the denominator of (4).

‡ At sufficiently low pressures the approach to optically thin radiation may involve Doppler broadening. The above arguments nevertheless suffice for order-of-magnitude considerations.

where $P$ is atmospheric pressure, with $\xi \ll 1$ denoting the limit of non-overlapping lines and $\xi \gg 1$ the limit of overlapping lines.

For present purposes we shall consider only the limit of non-overlapping lines, and for Lorentz lines this corresponds to the vicinity of the stratopause of Venus (Ramanathan & Cess 1974). In terms of the strong-line parameter, the divergence of the radiative flux $q$ applicable in the limit of non-overlapping lines is obtained by letting $\xi \to 0$ in equation (8) of Cess & Ramanathan (1972), such that

$$\frac{dq}{d\xi^{\frac{1}{2}}} = 2A_{01}\left[\alpha\mu^{\frac{1}{2}} \sum_{i=2}^{N} e_{\omega_i}(T_s)\left(\frac{2S_1 A_{0i}}{3S_1 A_{01}}\right)^{\frac{1}{2}} - e_{\omega_1}(T)\right], \tag{6}$$

where $e_{\omega_i}(T)$ is Planck's function at the wavenumber $\omega_i$ of the band centre, the subscript $i \geqslant 2$ refers to the near infra-red solar absorption bands of $CO_2$, $\alpha$ is the planetary angle factor, $\mu$ the cosine of the solar zenith angle and $T_s$ denotes the effective black-body temperature of the sun.

The absence of exchange integrals in (6) is consistent with discussions by Rodgers & Walshaw (1966), Dickinson (1972) and Ramanathan & Cess (1974). The form of (6) illustrates that the limit of strong non-overlapping lines is mathematically equivalent to optically thin radiation. But the equivalent 'mean absorption coefficient' will be much less than the Planck mean owing to line saturation. Putting it another way, in the strong-line limit the central portions of the lines are opaque and do not contribute to radiative transfer, while the far wings of the lines are optically thin and thus do not involve exchange integrals.

The radiative equilibrium temperature is determined from (6) by setting $dq/d\xi^{\frac{1}{2}} = 0$. Denoting this temperature by $T_e$, considering small departures from radiative equilibrium, employing Wien's approximation to Planck's function for $e_{\omega_1}(T)$ and assuming a hydrostatic atmosphere to describe $P(z)$ in (5), it readily follows from (6) that

$$\nabla . \mathbf{q} = (\rho C_p/\tau)(T - T_e), \tag{7}$$

where
$$\tau = \frac{C_p T_e}{1920 R e_{\omega_1}(T_e)}\left(\frac{Hd}{3S_1\gamma_0 A_{01}}\right)^{\frac{1}{2}}. \tag{8}$$

At the stratopause of Venus (Dickinson 1972), $T_e = 158\,°\mathrm{K}$ and $H = 3{\cdot}5\,\mathrm{km}$, while the $15\,\mu\mathrm{m}$ band parameters are summarized by Cess & Ramanathan (1972). We thus find that $\tau = 1{\cdot}7 \times 10^5\,\mathrm{s}$, and this is in excellent agreement with the result given by Dickinson (1972, figure 15) for the same altitude. Equation (8) illustrates that $\tau$ is independent of pressure, and this is consistent with the statement of Goody & Belton (1967) concerning strong non-overlapping Lorentz lines.

With the interpretation of $\tau$ as a radiative response time, (7) is of precisely the same form as a radiative heating approximation which has seen much recent application (e.g. Gierasch 1970; Gierasch & Sagan 1971; Stone 1972). This is important, since it illustrates that the response-time approximation is indeed applicable to atmospheric regions for which the rotational lines are Lorentzian and non-overlapping. With regard to the present stability problem, it would be tempting to extend (7) to lower altitudes by employing Dickinson's (1972) values for $\tau$. But Cess & Ramanathan (1973) have shown that, in the limit of overlapping lines ($\xi \gg 1$), (7) does not describe the local heating function as is necessary in a stability analysis. Instead (7) is useful only in a spatially averaged context. It would thus appear that for present purposes the application of (7) must be restricted to regions within the atmosphere for which the lines are non-overlapping; i.e. in the vicinity of the stratopause.

## 4. Stability results

Since the present radiative-transfer formulation is mathematically equivalent to the optically thin limit, we may employ directly the optically thin stability analysis of Dudis, but with a much smaller value for $G$ which accounts for line saturation. With $T' = T - T_E$, from (2) and (7), $G = 1/\tau S$. Following Dudis, we take the vertical shear to be $S = 0{\cdot}01\,\mathrm{s}^{-1}$. This value appears reasonable in the light of the interpretation of the Soviet Venera 7 data by Ainsworth & Herman (1972), who give $S \simeq 0{\cdot}02\,\mathrm{s}^{-1}$ within the upper troposphere; if the shear flow is driven by diurnal heating, one would anticipate a smaller value of $S$ at higher altitudes (Ramanathan 1973). With this value for $S$ together with $\tau = 1{\cdot}7 \times 10^5\,\mathrm{s}$, then $G = 6 \times 10^{-4}$ as compared with $G = 3{\cdot}5$ from Dudis.

Consider now the evaluation of $Ri_1$ and $Ri_2$ appropriate to the upper stratosphere of Venus. Dudis shows that, for $G = 0{\cdot}1$, $Ri_1$ is increased only slightly from the non-radiating value of $\frac{1}{4}$. Since the present value ($G = 6 \times 10^{-4}$) is more than two orders of magnitude less than $G = 0{\cdot}1$, it is reasonable to conclude that $Ri_1 = \frac{1}{4}$.

With regard to $Ri_2$, we shall employ the double limit $G/k$, $GRe \to \infty$ as treated by Dudis, where $k$ is the dimensionless wavenumber of the disturbance and $Re = SH^2/\nu$ is the Reynolds number of the shear layer, $\nu$ denoting the kinematic viscosity. We shall justify the use of this limit *a posteriori*. From Dudis

$$Ri_2 = 0{\cdot}53 G Re^{\frac{1}{3}}. \tag{9}$$

Employing previous values, $Re = 2{\cdot}5 \times 10^7$, and clearly $GRe \gg 1$. To determine whether $G/k \gg 1$, we need to estimate the value of $k$ corresponding to $Ri_2$. Dudis has tabulated $k$ as a function of $Re$ for $R \leqslant 10^4$. For higher Reynolds numbers we may invoke the additional limit $Re \to \infty$, but hold $kRe$ finite, thus retaining the viscous terms. This is equivalent to setting $k^2 = 0$ in equation (4.6) of Dudis, which shows that $kRe$ approaches a constant value for $Re \to \infty$.

We have not attempted to solve the resulting stability equation. Instead, from Dudis' numerical values $Rik^{\frac{1}{3}}G \simeq 2{\cdot}2$ for $1/Re \to 0$, so that from (9) $kRe \simeq 70$ for $Re \to \infty$, and we find that $G/k \simeq 200$, justifying use of the $G/k \to \infty$ limit. Equation (9) in turn yields $Ri_2 = 0{\cdot}093$. Since this is much less than $Ri_1$, the critical Richardson number is $Ri_1 = \frac{1}{4}$.

The above illustrates that radiative transfer will not play a significant role with regard to shear-flow stability for conditions representative of the upper stratosphere of Venus. The actual Richardson number for $S = 0{\cdot}01\,\mathrm{s}^{-1}$ and $dT/dz = 0$ is $Ri = 4{\cdot}9$, and this is considerably greater than the critical value.

This conclusion remains unchanged even if we arbitrarily select a shear-layer length scale which is less than the atmospheric scale height. For example, taking $L = 10\,\mathrm{m}$ ($L/H = 2{\cdot}9\times10^{-3} \ll 1$), $\delta \simeq 10^5$, so that the strong-line limit is still applicable, and if $H$ is replaced by $\frac{1}{2}L$ in (8) in accord with previous comments concerning Curtis–Godson scaling, then $G = 0{\cdot}008$ while $Re = 210$. From Dudis' results we again find that radiation will not significantly destabilize the shear flow.

We conclude that the apparent vertical mixing within the atmosphere of Venus (Dudis 1973) does not originate from radiative destabilization of a shear flow, at least within the framework of a linear stability analysis. Other possible explanations for vertical mixing have been proposed by Lindzen (1970), Matsuno (1970) and Hart (1972).

We thank V. Ramanathan for his constructive discussions. This work was supported by the National Science Foundation through Grant K036988.

*Note added in proof.* Recently Dudis (*J. Fluid Mech.* vol. 64, 1974, p. 65) has reconsidered the problem of stability within the stratosphere of Venus, employing a modified grey-gas differential (or Milne–Eddington) approximation to account for exchange integrals, and incorporating line saturation by estimating a response time from Goody & Belton (1967). While we disagree with this method for including exchange integrals, such exchange integrals are, as discussed herein, of no consequence within the upper stratosphere of Venus, and consequently this more recent study by Dudis is qualitatively consistent with our present results.

## REFERENCES

AINSWORTH, J. E. & HERMAN, J. R. 1972 *Goddard Space Flight Center Rep.* X-625-72-187.
CESS, R. D., MIGHDOLL, P. & TIWARI, S. N. 1967 *Int. J. Heat Mass Transfer*, **10**, 1521.
CESS, R. D. & RAMANATHAN, V. 1972 *J. Quant. Spectrosc. Radiat. Transfer*, **12**, 933.
CESS, R. D. & RAMANATHAN, V. 1973 *J. Quant. Spectrosc. Radiat. Transfer*, **13**, 79.
CESS, R. D. & TIWARI, S. N. 1972 *Advances in Heat Transfer*, vol. 8, p. 229. Academic.
DICKINSON, R. E. 1972 *J. Atmos. Sci.* **29**, 1531.
DUDIS, J. J. 1973 *J. Fluid Mech.* **58**, 81.
GIERASCH, P. J. 1970 *Icarus*, **13**, 25.
GIERASCH, P., GOODY, R. & STONE, P. 1970 *Geophys. Fluid Dyn.* **1**, 1.
GIERASCH, P. & SAGAN, C. 1971 *Icarus*, **14**, 312.
GOODY, R. M. & BELTON, M. J. S. 1967 *Planet. Space Sci.* **15**, 247.
HART, J. E. 1972 *J. Atmos. Sci.* **29**, 687.
LINDZEN, R. S. 1970 *J. Atmos. Sci.* **27**, 537.
MASLOWE, S. A. & THOMPSON, J. M. 1971 *Phys. Fluids*, **14**, 453.
MATSUNO, T. 1970 *J. Atmos. Sci.* **27**, 871.
RAMANATHAN, V. 1973 Ph.D. thesis, State University of New York at Stony Brook.
RAMANATHAN, V. & CESS, R. D. 1974 *Astrophys. J.* In Press.
RODGERS, C. D. & WALSHAW, C. D. 1966 *Quart. J. Roy. Met. Soc.* **92**, 67.
STONE, P. H. 1972 *J. Atmos. Sci.* **29**, 405.